$\therefore\ \triangle ABD = \triangle BCF + \triangle ACE.$ Q. E. D.

That the three lines AF, BE, and CD intersect in one point may be proved as follows:

Since EOB cuts the sides of $\triangle ACM$ in the points N, O, B, $CN\times OM\times AB = AN\times CO\times BM$....(1).

Since AOF cuts the sides of $\triangle BCM$ in O, P, F, $CO\times BP\times AM = OM\times CP\times AP$....(2).

Multiplying (1) by (2) and canceling, $CN\times BP\times AM = AN\times BM\times CP$.

Also demonstrated by *G. B. M. ZERR*, and the *PROPOSER*.

142. Proposed by WILLIAM HOOVER, A.M., Ph.D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Show that an infinite number of triangles can be inscribed in $x^2/a^2+y^2/b^2-1=0$ whose sides touch $a^2x^2+b^2y^2=\dfrac{a^4b^4}{(a^2+b^2)^2}$.

I. Solution by the PROPOSER.

The curves are $b^2x^2+a^2y^2-a^2b^2=0$....(1),

$$\text{and } a^2(a^2+b^2)^2x^2+b^2(a^2+b^2)^2y^2-a^4b^4=0\text{....(2).}$$

The invariants of (1) are $\triangle=-a^4b^4$,

$$\theta'=-a^2b^2(a^4+a^2b^2+b^4)^2$$

and of (2), $\triangle=-a^6b^6(a^2+b^2)^4$,

$$\theta'=-2a^4b^4(a^2+b^2)^2(a^4+a^2b^2+b^4).$$

By the usual theory, the conditions of the problem are fulfilled if $\theta^2=4\triangle\theta'$....(3), which is easily seen to be the case here.

II. Solution by GEORGE A. OSBORNE, Professor of Mathematics, Massachusetts Institute of Technology, Boston, Mass.

The problem is a special case of the following:

An infinite number of triangles can be inscribed in the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ whose sides touch $\dfrac{x^2}{a'^2}+\dfrac{y^2}{b'^2}=1$, provided $\dfrac{a'}{a}+\dfrac{b'}{b}=1$.

The problem is considered in Salmon's *Conic Sections*, Art. 376, page 342, for the general form of the conic.

$$S=ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0.$$

$$S'=a'x^2+b'y^2+c'z^2+2f'yz+5g'zx+2h'xy=0.$$

The condition that an infinite number of triangles may be inscribed in S and circumscribed about S', is $\Theta'^2=4\Delta'\Theta$....(1).

Δ, Δ', are the discriminants of S, S'.

$$\Delta=\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}, \quad \Delta'=\begin{vmatrix} a' & h' & g' \\ h' & b' & f' \\ g' & f' & c' \end{vmatrix}.$$

$$\Theta=Aa'+Bb'+Cc'+2Ff'+2Gg'+2Hh',$$

$$\Theta'=A'a+B'b+C'c+2F'f+2G'g+2H'h,$$

where A, B, C, etc., are minors of Δ, and A', B', C', etc., are minors of Δ'.

For $\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0$ and $\frac{x^2}{a'^2}+\frac{y^2}{b'^2}-1=0$, we find $\Delta'=-\frac{1}{a'^2b'^2}$,

$$\Theta=-\frac{1}{a^2\,b'^2}-\frac{1}{a'^2\,b^2}-\frac{1}{a^2\,b^2}, \quad \Theta'=-\frac{1}{a'^2\,b^2}-\frac{1}{a^2\,b'^2}-\frac{1}{a'^2b'^2}.$$

Substituting in (1), we have

$$\left(\frac{a'^2}{a^2}+\frac{b'^2}{b^2}+1\right)^2=4\left(\frac{a'^2}{a^2}+\frac{b'^2}{b^2}+\frac{a'^2b'^2}{a^2\,b^2}\right)$$

from which $\frac{a'}{a}\pm\frac{b'}{b}=\pm1$.

The only real solution for ellipses is

$$\frac{a'}{a}+\frac{b'}{b}=1.$$

In problem 142,

$$a'=\frac{ab^2}{a^2+b^2},\ b'=\frac{a^2b}{a^2+b^2},\ \text{from which}\ \frac{a'}{a}+\frac{b'}{b}=1.$$

A very excellent demonstration was received from *G. B. M. ZERR.*

CALCULUS.

NOTE ON CENTER OF CURVATURE.

By GEORGE R. DEAN, A. M., Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

The fact, that the point of intersection of two normals which approach each other is neither at infinity nor at the foot of the normals when they become coincident, is not plain to most students. The difficulty may be overcome, in some instances, in the following manner.

Let (x_1, y_1), (x_2, y_2) be two points of the curve ; m_1; m_2 the slope of the tangents at these points. The equations of the normals are then